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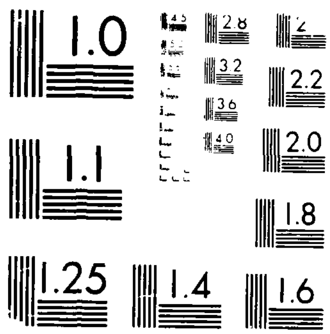

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Applications of tools from group theory and nonlinear analysis to global bifurcation problems from solid mechanics are summarized. These include both topological and computational approaches for problems involving structural frameworks, strings, rods and 3-dimensional elastic bodies.

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Symmetry and Global Bifurcation in Nonlinear Solid Mechanics

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1. Introduction.

The purpose of this investigation was to develop and implement group-theoretic strategies in nonlinear problems of solid mechanics. In contrast to the well-cultivated subject of local equivariant bifurcation theory [1], [2], [3], this study was focused on global analysis in the presence of symmetry. Several projects have been completed. Since the effective date of this grant, August 1, 1986, papers [H5]-[H8][1] have been written, papers [H9]-[H11] are nearly completed, and substantial progress has been made on papers [H12]-[H15]. A description of the major results is given in the next section.

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[1] References prefaced with "H" are listed in Section 4. General references are in Section 3.

2. Description of Results

Paper [H5] investigates the foundations of equivariance in continuum mechanics in the specific context of nonlinear elastostatics. Well known techniques for exploiting equivariance (in bifurcation problems) are typically stated with mathematical precision, cf. [1], [2], [3], e.g. In contrast, the fundamental ingredients of equivariance in physical problems have either been ignored or vaguely attributed to "frame indifference," in the literature. In [H5] it is shown that the symmetry of the reference configuration, the global material symmetry of the entire body, the principle of material frame indifference, and the symmetry of the loading and boundary data each play a crucial role in the equivariance of the field equations.

Novel features of the work include a definition of global material symmetry for general nonlinearly elastic bodies (as compared with the conventional notion of material symmetry at a point) and a technique for obtaining reduced problems having solutions with prescribed symmetries. A nontrivial application to general aelotropic bodies of revolution (e.g., tree trunks and radially and/or circumferentially reinforced bodies) is presented.

Paper [H6] presents tools for exploiting symmetry in numerical (static) bifurcation problems. The methodology is tailored for use with well known path-following techniques. The basic theme is the construction of reduced problems corresponding to the symmetry group and its subgroups. As demonstrated in the analysis of a hexagonal lattice structure (cf. [H6] §3), the approach often drastically reduces the number of unknowns, simplifies the analysis and accurate computation of symmetry-breaking bifurcation points.

reduces multiple symmetry-breaking bifurcations to simple bifurcation problems, and enables the automatic generation of families of solution branches from the computation of a single family member. Admittedly, most of the numerical computations were carried out prior to the time of this grant [H2]. However, those efforts required a great deal of guesswork and intuition, since the "reduced problem" was only understood there in principle (as reported in Theorem 3.1 of [H6]). The explicit construction (3.4)-(3.11) and (3.34)-(3.35) and Theorem 3.3 (which generalizes the equivariant branching lemma [3] to include the case of bifurcation from a nontrivial solution path) of [H6] are new contributions. Both are essential to justifying the numerics of [H2] (as presented in [H6]) and to extending the methodology to a broad class of potential applications.

In paper [H7] it is shown that the exploitation of symmetry as a first step in the analysis of a bifurcation problem can deliver a reduced problem amenable to global (as well as local) analysis. This is in contrast to the usual approach of exploiting the equivariance of the local bifurcation equations, cf. [1], [2], [3], e.g. The application of well established continuation theorems [4], [5] to a reduced problem demonstrates that symmetry is preserved along global continua of solutions. The approach is applied to the problem of large post-buckling of a nonlinearly elastic circular ring under uniform hydrostatic pressure, and several new results are obtained. Specific symmetries of global bifurcating solution branches are enumerated, which enables a detailed qualitative analysis. In particular, a specific nodal pattern can be identified, and thus symmetry-breaking secondary bifurcation is shown not to occur.

Paper [HS] presents a practical numerical method for computing the symmetry modes of structures with symmetry. The basis vectors (symmetry modes) needed to construct a reduced problem are the eigenvectors associated with the multiple unit eigenvalue of a well-defined projection matrix, cf. [HS, (6)]. The multiplicity is also well-defined, cf. [H8, (7)]. Thus, the symmetry modes can be extracted by a combination of matrix multiplication and deflation. Several examples are provided in an attempt to communicate the utility of the reduction procedure to structural engineers. The illustrations range from the pedagogical to the nontrivial, including a case where the loading possesses less symmetry than the structure.

Paper [H9] presents a technique for the computation of symmetry-breaking bifurcation points (along one-parameter solution branches) and demonstrates that the same algorithm can be used to compute paths of symmetry-breaking bifurcation points for two-parameter problems. The former topic has been considered by other investigators [6], [7] for the special case of simple symmetry-breaking bifurcation points. In contrast, the approach in [H9] is based on the equivariant branching lemma, and is applicable whenever a multiple bifurcation problem can be reduced to a simple one. Moreover, the extended system of [6], [7] essentially doubles the computer storage requirements. In [H9] the calculation of the eigenvector via inverse iteration is effectively divorced from a novel extended system, which requires a minimal increase in storage size. The technique is illustrated in the computation of several paths of symmetry-breaking bifurcation points for a space structure subjected to two independent loadings.

Paper [H10] presents a global analysis of a simple two-bar structure made of a general nonlinearly elastic material under dead load. The detailed qualitative results, which include global bifurcation from a nontrivial (primary) solution path, depend crucially upon the Z_2 symmetries of the governing equations, with group actions on both the dependent and independent variables. The dependence of the entire solution set on two parameters (load and aspect ratio) is examined. The most important conclusion of the paper is that the use of a popular constitutive law in this model [H3] fails to capture generic solution branches. This has important ramifications for the analysis of more complex structures.

Paper [H11][2] re-examines some of the classical linear problems of structural analysis for systems with symmetry. Aside from the intrinsic importance of the linear theory in engineering practice, the solutions of linearized equations and linearized eigenvalue problems are required repeatedly in path-following algorithms for nonlinear problems [S]. Thus, the connection of [H11] with [H6] and [H9] is clear. The use of group theory in linear problems from physics and chemistry is well known, cf. [9], [10], e.g. The basic idea is to split the full problem into several smaller, decoupled problems, the solutions of which are easily obtained. Such an approach has apparently gone unnoticed in the field of structural analysis, and the purpose of [H11] is to demonstrate its utility in a computational setting.

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[2]This project was also supported in part by AFOSR/URI Nonlinear Dyn. & Control of Flexible Structures F4-9620-S7-C-0011

However, the treatment is not completely pedagogical, as several new results are obtained. First, it is shown that the required projections (from group representation theory) can be carried out efficiently on the element level. Thus, the reduced problems can be constructed directly without ever assembling the full problem. In the context of examples, it is also demonstrated that multiple eigenvalues can often be eliminated. Finally, it is shown that the group analysis of bending degrees of freedom (dof) is dual in some sense, to that of translational dof. This leads to the fact that no further group-theoretic work is required for the bending dof of a frame once the group analysis of the translational dof has been completed.

In paper [H12], the approach introduced in [H7] is brought to bear on a global multiparameter bifurcation problem, the solutions of which correspond to steady-state configurations of an initially straight, whirling, nonlinearly elastic wire that carries a current in a uniform magnetic field. The same problem has been studied by another investigator [12], who also considered the case without whirling in an earlier work [11]. The latter problem admits closed-form expressions for the global bifurcating branches. Such is not the case in the presence of whirling. As pointed out in [12], the local analysis of bifurcation is intractable by standard techniques, due to the SO(2) (rotational) symmetry in the problem. The entire development in [12] is devoted to the resolution of the local problem by variational techniques.

In [H12] it is shown that the exploitation of symmetry yields a standard one-dimensional bifurcation problem, amenable to both local and global analysis. However, the obvious identification of SO(2) symmetry is

not enough to simplify things; the only isotropy subgroup of SO(2) that leaves a null vector (of the linearized problem) invariant is the singleton $\{1\} \subset SO(2)$, where "1" denotes the identity. Thus, no reduction seems possible. In fact, the complete symmetry group of the problem is $SO(2) \times C_2 \subset SO(3)$. The reduction associated with the isotropy subgroup $\{1\} \times C_2 \subset SO(2) \times C_2$ delivers the simple bifurcation problem. The existence of connected, global bifurcating solution sheets of (Lebesque) dimension ≥ 2 is readily established by a well known approach [13]. Moreover, it is shown that the bifurcating sheets are unbounded, by the identification of a distinct nodal character along each branch.

The techniques introduced in [H6] and [H9] are further illustrated in the analysis of a geodesic dome structure in paper [H13]. The reduction of the 111 degree-of-freedom structure to a twelve-dimensional system (for the computation of the primary solution path) is striking. However, the accurate determination of numerous bifurcation points and the computation of the subsequent secondary branches are the most noteworthy aspects of the paper. The primary path contains "clusters" of closely spaced singular points, thus rendering the full problem extremely difficult if not intractable for any standard path-following method. However, the distinct symmetry-breaking character of each bifurcation point enables its accurate computation by the method presented in [H9].

The planar bending of an incompressible elastic block into a circular sector by end moments is a classical problem with a well-known exact solution [14]. Paper [H14] treats the same problem for the case of a general compressible hyperelastic material. The semi-inverse method (which is an

obvious exploitation of $Z_2 \times Z_2$ symmetry in this case) delivers a two-point boundary-value problem in terms of dependent variables that are related to the principal stretches within the plane of bending. In spite of the existence of a first integral, a qualitative analysis is difficult at this stage. However, a transformation in favor of dependent variables corresponding to a principal stress component and a principal stretch component yields two coupled, first-order differential equations that are amenable to a phase-plane analysis. With reasonable conditions on the stored energy function, the existence and uniqueness of the solution (within the class of trial solutions that satisfy the semi-inverse assumptions) is established.

Paper [H15] considers the problem of a thin, infinite, incompressible, elastic layer loaded with equal and opposite uniform, dead-load tractions perpendicular to the undeformed boundaries, cf. Fig. 1.

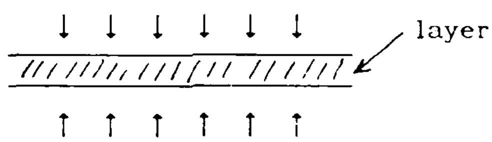

Figure 1

The mathematical approach to this problem is well motivated by the work of Sattinger [1] and Golubitsky, et. al., [15] on the Bénard problem. However, the physics here is completely different; we consider the static equilibrium of a solid. Moreover, unlike the Bénard problem, which is a classical problem motivated by numerous experiments, we know of no experiments on

elastic layers. Hence, we hope that our theoretical results will motivate experimental work.

The problem admits an obvious trivial solution - no deformation accompanied by an adjustment of the internal pressure to the level of the external traction. Of course, this presumes that the layer is immobile at infinity. The linearized problem (about the trivial solution) has an infinite-dimensional kernel, which is due to the presence of the full 2-dimensional Euclidean group. The problem has additional Z_2 symmetry about the midplane of the layer. Following the approach in [15], we first restrict solutions to the class of all doubly-periodic functions on a hexagonal lattice, which reduces the symmetry group to $D_6 \times T_2 \times Z_2$. However, unlike the approach in [15], we do not proceed to the (local) bifurcation equations at this stage. Rather, we follow [H5] and [H7] by restricting the system of p.d.e.'s to the isotropy subgroups of $D_6 \times T_2 \times Z_2$. This delivers reduced problems with linearizations that are readily shown to be Fredholm of index zero, which in turn leads to standard one-dimensional bifurcation problems. In this way we obtain the mathematical analogues of the results for the Bénard problem [15] - distinct equilibrium states with patterns comprising rectangles, triangles, hexagons and rolls.

Various projects proposed in 1985 have been completed. The proposed work on structural frameworks has led to [H6, H8, H10, H11, H13]. The proposed project on the statics of planar rings led to [H7]. Moreover, the research was drawn naturally into new directions not originally contemplated, but in the spirit of the proposal [H9, H12, H14, H15]. Some promising

preliminary results on the forced vibration of planar rings and on the statics of nonplanar rings subjected to twisting moments will be reported in a forthcoming proposal. The treatment of nonaxisymmetric buckling of shells of revolution is only now beginning. The proposed work on planar rings with D_n symmetry has been dropped in view of a recent work [16], where the apparent dicotomy between systems having the symmetries D_n as $n \rightarrow \infty$ and $O(2)$ is investigated. The subtleties involved in establishing equivariance in rod models is indicated in [H7], but little has been done in establishing the foundations of equivariance for general rod and shell models. The investigation concerning the equivariance of general finite-element equations in nonlinear elasticity remains to be carried out, although the experience gained in [H5] and [H6] implies that symmetry should be exploited prior to a discretization. This allows for adaptive remeshing along different branches of solutions with distinct symmetries. Further studies on the equivariance of the weak form of the equations of elastostatics with and without constraints (rod and shell equations and finite-element equations are special cases of the former) will also be described in a forthcoming proposal.

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[H7] "Global Bifurcation and Continuation in the Presence of Symmetry with an Application to Solid Mechanics," to appear SIAM Jour. Math. Anal.

[H8] "Computation of Symmetry Modes and Exact Reduction in Nonlinear Structural Analysis," (with P. Chang), to appear Computers and Structures.

[H9] "Computation of Paths of Symmetry Breaking Bifurcation Points," in preparation.

[H10] "A Paradigm of Bifurcation and Some Pathologies of a Popular Constitutive Law," in preparation.

[H11] "Linear Analysis of Skeletal Structures with Symmetry. A Group Theoretic Approach" (with J. Treacy), in preparation.

[H12] "Large Buckled States of a Whirling Conducting Elastic Wire in a Magnetic Field: Global Multiparameter Bifurcation in the Presence of Symmetry," in preparation.

[H13] "Bifurcation and Instability of a Geodesic Dome" (with P. Chang), in preparation.

[H14] "Pure Bending of Compressible Hyperelastic Blocks," (with A. Szeri), in preparation.

[H15] "Pattern Selection in a Compressed Elastic Layer," (with Y.-C. Chen), in preparation.

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